



# Beyond dynamical mean-field theory of neural networks

Massimiliano Muratori, Bruno Cessac

## ► To cite this version:

Massimiliano Muratori, Bruno Cessac. Beyond dynamical mean-field theory of neural networks. Twenty Second Annual Computational Neuroscience Meeting: CNS 2013, Jul 2013, Paris, France. 14 (Suppl 1), pp.P60, 2013. hal-00842309

**HAL Id: hal-00842309**

**<https://hal.inria.fr/hal-00842309>**

Submitted on 8 Jul 2013

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

POSTER PRESENTATION

Open Access

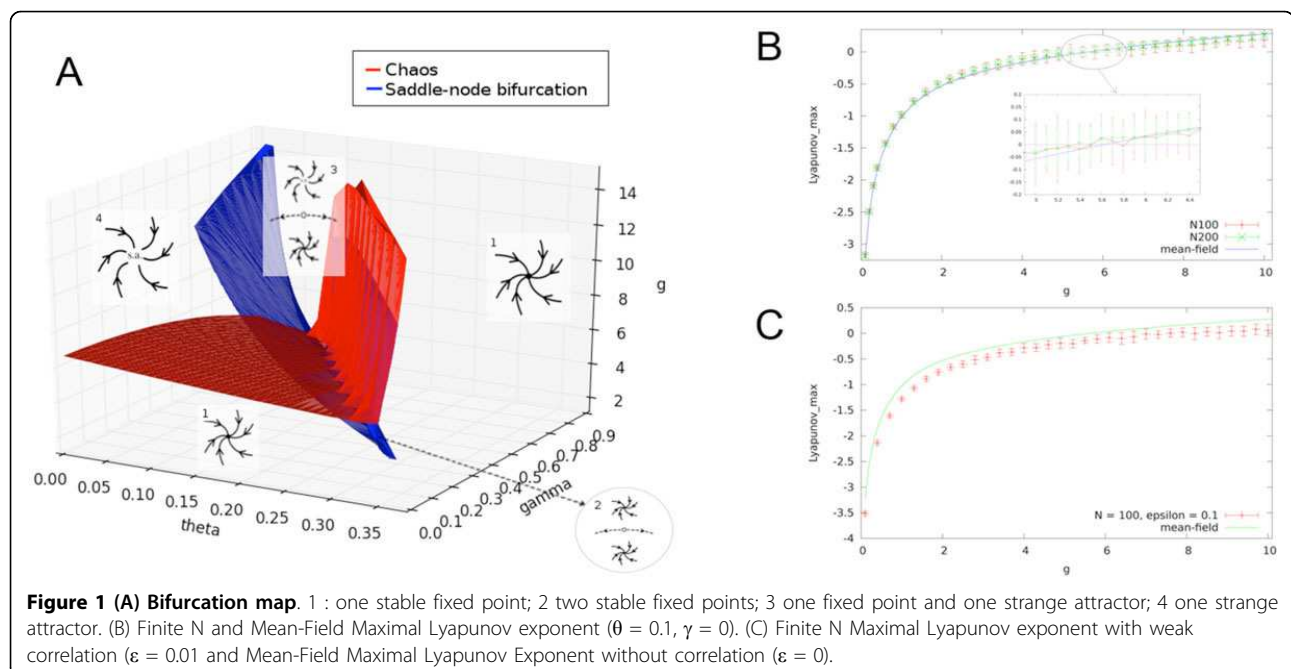
# Beyond dynamical mean-field theory of neural networks

Massimiliano Muratori\*, Bruno Cessac

From Twenty Second Annual Computational Neuroscience Meeting: CNS\*2013  
Paris, France. 13-18 July 2013

We consider a set of  $N$  firing rate neurons with discrete time dynamics and a leak term  $\gamma$ . The nonlinearity of the sigmoid is controlled by a parameter  $g$  and each neuron has a firing threshold  $\theta$ , Gaussian distributed (thresholds are uncorrelated). The network is fully connected with correlated Gaussian random synaptic weights, with mean zero and covariance matrix  $C/N$ . When synaptic weights are uncorrelated the dynamic mean field theory developed in [1-3] allows us to draw the bifurcation diagram of the model in the thermodynamic limit ( $N$  tending to infinity): in particular there is sharp transition from fixed point to

chaos characterized by the maximum Lyapunov exponent, which is known analytically in the thermodynamic limit. The bifurcation diagram is drawn in Figure 1 A. However, mean-field theory is exact only in the thermodynamic limit and when synaptic weights are uncorrelated. What are the deviations from mean-field theory observed when one departs from these hypotheses? We have first studied the finite size dynamics. For finite  $N$  the maximal Lyapunov exponent has a plateau at 0 corresponding to a transition to chaos by quasi-periodicity where dynamics is at the edge of chaos (Figure 1 B). This plateau disappears in the



thermodynamic limit. Thus, mean-field theory neglects an important finite-sized effect since neuronal dynamics at the edge of chaos has strong implications on learning performances of the network [4]. We also studied the effect of a weak correlation (of amplitude  $\epsilon$ ) on dynamics. Even, when  $\epsilon$  is small one detects an important deviation on the maximal Lyapunov exponent (Figure 1 C).

#### Acknowledgements

This work was supported by INRIA, ERC-NERVI number 227747, KEOPS ANR-CONICYT and European Union Project # FP7-269921 (BrainScales), Renvision grant agreement N 600847 and Mathemacis FP7-ICT\_2011.9.7.

Published: 8 July 2013

#### References

1. Cessac B, Doyon B, Quoy M, Samuelides M: **Mean-field equations, bifurcation map and route to chaos in discrete time neural networks.** *Physica D* 1994, **74**:24-44.
2. Cessac B: **Increase in Complexity in Random Neural Networks.** *J Phys I France* 1995, **5**:409-432.
3. Moynot O, Samuelides M: **Large deviations and mean-field theory for asymmetric random recurrent neural networks.** *Probability Theory and Related Fields* 2002, **123**:41-75, Springer-Verlag.
4. Legenstein R, Maass W: **Edge of Chaos and Prediction of Computational Performance for Neural Circuit Models.** *Neural Networks* 2007, **20**:323-334.

doi:10.1186/1471-2202-14-S1-P60

**Cite this article as:** Muratori and Cessac: **Beyond dynamical mean-field theory of neural networks.** *BMC Neuroscience* 2013 **14**(Suppl 1):P60.

**Submit your next manuscript to BioMed Central and take full advantage of:**

- Convenient online submission
- Thorough peer review
- No space constraints or color figure charges
- Immediate publication on acceptance
- Inclusion in PubMed, CAS, Scopus and Google Scholar
- Research which is freely available for redistribution

Submit your manuscript at  
[www.biomedcentral.com/submit](http://www.biomedcentral.com/submit)

